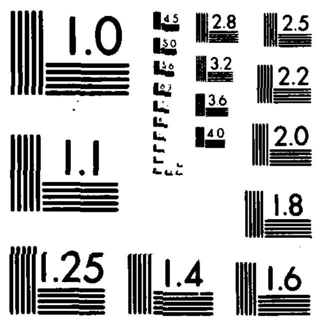
AD-A083 987   PENNSYLVANIA STATE UNIV UNIVERSITY PARK APPLIED RESE--ETC F/G 21/5
DOCUMENTATION OF CASCADE UNSTEADY PRESSURE DIFFERENCE PROGRAM.(U)
JAN 80 I SHEN   N00024-79-C-6043
UNCLASSIFIED   ARL/PSU/TM-80-10   NL

END
DATE
FILMED
6 80
DTIC


1.0
4.5
5.0
5.6
6.3
2.8
3.2
3.6
4.0
2.5
2.2
2.0
1.1
1.8
1.25
1.4
1.6

(12)

LEVEL

DOCUMENTATION OF CASCADE UNSTEADY PRESSURE DIFFERENCE PROGRAM

I-C, Shen

Technical Memorandum
File No. 80-10
January 5, 1980
Contract No. N00024-79-C6043

Copy No. 25

The Pennsylvania State University
APPLIED RESEARCH LABORATORY
Post Office Box 30
State College, PA 16801

Approved for Public Release
Distribution Unlimited

NAVY DEPARTMENT

NAVAL SEA SYSTEMS COMMAND

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
TM 80-10

2. GOVT ACCESSION NO.
14

3. RECIPIENT'S CATALOG NUMBER
ARL/PSU/TM-80-10

4. TITLE (and Subtitle)
6
DOCUMENTATION OF CASCADE UNSTEADY PRESSURE DIFFERENCE PROGRAM

5. TYPE OF REPORT & PERIOD COVERED
9
Technical Memorandum

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
10
I-C. Shen

8. CONTRACT OR GRANT NUMBER(s)
15
N00024-79-C-6043

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Applied Research Laboratory
P. O. Box 30
~~State College~~, PA 16801

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
12 46

11. CONTROLLING OFFICE NAME AND ADDRESS
Naval Sea Systems Command
Code NSEA-063R, Department of the Navy
Washington, DC 20362

12. REPORT DATE
11
Jan[redacted]80

13. NUMBER OF PAGES
42

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

15. SECURITY CLASS. (of this report)
UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)
Approved for Public Release. Distribution Unlimited.
Per NAVSEA - May 1, 1980.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
axial, flow, turbomachinery, unsteady, computer, program

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The response of an axial-flow turbomachine to spatial velocity variations can be determined by the unsteady pressure distribution function which has been developed and represents an extension of an earlier analysis of the unsteady lift. A computer program has been written to formulate the unsteady pressure difference as a function of design parameters. This program is versatile in that it includes the effects of the cascade geometrical parameters and disturbance flow characterisitics, such as the blade camber, angle of (continuing)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED 391007

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

incidence, nonconvected disturbance, and it also gives the unsteady lift and moment. The documentation and the use of this program are presented herein.

Subject: Documentation of Cascade Unsteady Pressure Difference Program

References: See page 20

Abstract: The unsteady response of an axial-flow turbomachine to spatial velocity variations can be determined by the unsteady pressure distribution function which has been developed and represents an extension of an earlier analysis of the unsteady lift. A computer program has been written to formulate the unsteady pressure difference as a function of design parameters. This program is versatile in that it includes the effects of the cascade geometrical parameters and disturbance flow characteristics, such as the blade camber, angle of incidence, nonconvected disturbance, and it also gives the unsteady lift and moment. The documentation and the use of this program are presented herein.

Acknowledgment: This work was sponsored by the U. S. Navy, Naval Sea Systems Command, Code NSEA-063R.

Table of Contents

List of Tables

List of Figures

List of Symbols

Symbol	Definition
a	cascade parameter, $a = \frac{ce^{-i\xi}}{2s}$
b	cascade parameter, $b = \frac{ce^{i\xi}}{2s}$
B_1	cascade wake integral, $B_1 = \int_1^\infty \frac{e^{-i\omega\lambda^+}}{\sqrt{g_\lambda^2 - 1}} d\lambda^+$
B_2	cascade wake integral, $B_2 = \int_1^\infty \frac{e^{-\omega\lambda^+}}{\sqrt{h_\lambda^2 - 1}} d\lambda^+$
c	blade chord
$\Delta\tilde{C}_p$	coefficient of unsteady pressure difference, $\Delta\tilde{C}_p = \frac{\overline{\Delta p}(\sigma^+)}{\rho W_m \hat{w}_d}$
C_1	cascade function, $C_1 = \sqrt{\frac{g_c + 1}{g_c - 1}}$
C_2	cascade function, $C_2 = \sqrt{\frac{h_c + 1}{h_c - 1}}$
$C(\omega)$	Theodorsen function
$\tilde{C}_L$	unsteady lift coefficient, $\frac{\tilde{L}}{\pi c \rho W_m \hat{w}_d e^{i\nu t}}$
$\tilde{C}_M$	unsteady pitching moment coefficient, $\frac{\tilde{M}}{\pi c^2 \rho W_m \hat{w}_d e^{i\nu t}}$

Symbol	Definition
D_1	cascade wake integral, $D_1 = \int_1^\infty \left[\sqrt{\frac{g_\lambda + 1}{g_\lambda - 1}} - 1\right] e^{-i\omega\lambda^+} d\lambda^+$
D_2	cascade wake integral, $D_2 = \int_1^\infty \left[\sqrt{\frac{h_\lambda + 1}{h_\lambda - 1}} - 1\right] e^{-i\omega\lambda} d\lambda^+$
E_1	cascade function, $E_1 = \frac{1}{g_c - \sigma^+} \sqrt{\frac{g_c + 1}{g_c - 1}}$
E_2	cascade function, $E_2 = \frac{1}{h_c - \sigma^+} \sqrt{\frac{h_c + 1}{h_c - 1}}$
g_c	cascade vortex function, $g_c = x_c^+ + \frac{n}{ia}$
g_λ	cascade wake function, $g_\lambda = \lambda^+ + \frac{n}{ia}$
g_1	cascade wake function evaluated at $\lambda^+ = 1$, $g_1 = 1 + \frac{n}{ia}$
h_c	cascade vortex function, $h_c = x_c^+ - \frac{n}{ib}$
h_λ	cascade wake function, $h_\lambda = \lambda^+ - \frac{n}{ib}$
h_1	cascade wake function evaluated at $\lambda^+ = 1$, $h_1 = 1 - \frac{n}{ib}$
$H_n^{(2)}(\omega)$	Hankel function of the second kind of order n

Symbol	Definition
$J_n(\lambda)$	Bessel function of the first kind of order n
ℓ	wavelength of disturbance
ℓ_c	wavelength along blade chord
L	lift force
M	pitching moment about the midchord, positive for leading-edge up rotation
M_c	cascade influence function,

$$M_c = \left[\sum_{-\infty}^{-1} + \sum_{1}^{\infty}\right] e^{in\tau} [C_1 + C_2 - 2] - i\omega e^{i\omega} \left[\sum_{-\infty}^{-1} + \sum_{1}^{\infty}\right] e^{in\tau} [D_1 + D_2]$$

Symbol	Definition
n	cascade blade number, n = 0 for reference blade
p	pressure
Δp	pressure difference, $\Delta p = p_{(-)} - p_{(+)}$
r	radial distance, dummy variable
s	blade-to-blade spacing in cascade
u	chordwise velocity component
U	rotor blade rotational velocity
v	transverse velocity component
V	stator exit or circumferential-mean rotor inlet absolute velocity
w_d	magnitude of disturbance velocity
W	blade relative velocity
W_1	circumferential-mean rotor inlet relative velocity
W_2	circumferential-mean rotor exit velocity
x	chordwise coordinate on reference blade
y	transverse coordinate on reference blade

Symbol	Definition
y_m	maximum blade camber
$Z(\lambda,\sigma^+)$	function in the equation of unsteady pressure distribution for an isolated airfoil, $Z(\lambda,\sigma^+) = \sum_{n=1}^{\infty} i^n J_n(\lambda) \sin [n(\pi - \cos^{-1} \sigma^+)]$
α	blade inlet incidence angle
β	relative flow angle measured from axial direction
γ	vorticity
Γ	circulation
Δ	circulation coefficient, $\Delta = \frac{\overline{\Gamma} e^{i\omega}}{c}$
ε	disturbance flow angle measured from the y-direction
λ	generalized reduced frequency or dummy variable
$\Lambda(\sigma,r)$	function, $\Lambda(\sigma,r) = 2 \tan^{-1} \left[\sqrt{\frac{1 - \sigma}{1 + \sigma} \frac{r + 1}{r - 1}}\right] - \pi$
ν	frequency
ξ	stagger angle of cascade measured from axial direction
ρ	mass density of fluid
σ	arbitrary point on reference blade
τ	intra-blade phase angle, $\tau = - \frac{2\pi s}{\ell}$
ϕ	phase angle
Φ	angle defined by $\Phi = \pi - \beta - \xi$
ω	reduced frequency

Symbol	Definition
SUBSCRIPT	
c	blade quarter-chord point
C.P.	center of pressure
d	disturbance
I	imaginary part of a complex quantity
L	related to lift
m	circumferential mean value
M	related to moment
n	blade number in cascade, $n = 0, \pm 1, \pm 2, \pm 3$
o	quantity related to reference blade
p	point on the reference blade
r	dummy variable represents c or λ
R	real part of a complex quantity
s	steady
u	unsteady
w	wake
λ	arbitrary point in the wake
(+)	pressure side
(-)	suction side
SUPERSCRIPT	
+	quantities expressed in nondimensional coordinate system
-	amplitude of a harmonic function in time
^	amplitude of a harmonic function of position and time
~	unsteady

INTRODUCTION

The presence of pressure fluctuations on a turbomachine blade row is known to cause undesirable effects, such as vibration, radiated noise, and performance degradation. It is important, therefore, to provide a means by which a turbomachine designer can predict the unsteady response of a blade row and select proper design parameters to minimize these effects.

The present analysis represents an alternative method, which is derived from the previous theory developed by Henderson [1], to determine the unsteady lift in a turbomachine blade row. The flow is assumed two-dimensional, incompressible, and inviscid. The vortex representation of the unsteady cascade flow is shown in Figure 2 in Reference 1. An explicit solution has been derived for the unsteady pressure difference on the surface of a turbomachine blade row operating in a spatially varying disturbance flow field, Figure 1. Specifically, this solution is expressed in terms of the cascade geometrical parameters—space-chord ratio, s/c, stagger angle, ξ, blade camber, y_m, mean angle of incidence, α_m, and the disturbance flow characteristics, represented by the reduced frequencies, ω and λ.

One of the advantages of employing the present analysis is that the parameters of unsteady lift and pitching moment can be easily obtained by direct numerical integration of the unsteady pressure difference across the airfoil chord.

A program has been written to formulate the unsteady pressure distribution on a single blade of a blade row as a function of the design parameters and it performs an integration of the pressures to obtain the unsteady lift and pitching moment using a quadrature method.

The significance of the present program can be summarized as follows:

1. Because the effect of blade-to-blade interaction is considered, the

resonance phenomena which occurs when the blade spacing equals the disturbance wavelength can be predicted.

2. The effect of the blade camber in a cascade of airfoils is included.
3. The effect of the mean incidence angle of the airfoils is included.
4. The program accounts for either a nonconvected or convected disturbance.
5. The program gives a complete description of the unsteady response, i.e., the unsteady pressure distribution, lift, moment, and center-of-pressure location.

DEFINITION OF THE PROBLEM

The problem considered is the unsteady pressure distribution on the blades of a cascade when the blade row experiences a spatial variation in the inflow; for example, the nonuniform flow caused by the wakes of an upstream blade row. The flow field is assumed to be two-dimensional, inviscid, and incompressible, thus representing the development of a cylindrical surface as shown in Figure 1. This represents the passage of a rotor through the wakes of an upstream stator blade with a swirling mean flow. These wakes have a maximum velocity deficit of $2w_d$ and are transported over the rotor by the velocity, W_m, the circumferential-mean velocity relative to the rotor blades with the wake present. The velocity deficit, w_d, represents the fluctuation about the mean velocity, W_m.

The description of the wake deficit shown in Figure 1 can be accomplished by using Fourier series representation. From this representation, the contribution of each harmonic of the velocity variation to the unsteady response of the blade can be determined. The resulting components of the disturbance velocity, w_d, normal and parallel to the chord can be expressed from the geometry of Figure 1 as,

$$\begin{aligned} v_d &= - w_d \sin \Phi \\ u_d &= - w_d \cos \Phi \end{aligned} \tag{1}$$

The consideration of the disturbance w_d as components parallel and normal to the chord facilitates the solution of this problem and it is possible to add the respective solutions since the model employed is linear.

FORMULATION OF THE UNSTEADY PRESSURE DISTRIBUTION

In order to generate values of the coefficients of unsteady pressure difference, $\Delta\tilde{C}_p$, unsteady lift, $\tilde{C}_L$, and unsteady moment, $\tilde{C}_M$, it is necessary to consider the contribution of blade geometry to this unsteady response. Starting from Equation (A20) in Reference 1, and substituting the boundary conditions, Equation (14), into this equation, the function of unsteady pressure distribution can be determined and is expressed in an explicit form as:

$$\begin{aligned}
\Delta\tilde{C}_p = {} & 4\, y_m^{+} \hat{u}_d \left\{ -\sqrt{\frac{1-\sigma^+}{1+\sigma^+}}\, i\, J_1(\lambda) - i\,\frac{2\omega}{\lambda^2} Z(\lambda,\sigma^+) \right. \\
& \left. + \left(\frac{\omega}{\lambda} - 1\right)\left[2\sigma^+ Z(\lambda,\sigma^+) - \sqrt{1-\sigma^{+2}}\, J_o(\lambda)\right]\right\} \\
& + 2\,\hat{v}_d \left\{ \sqrt{\frac{1-\sigma^+}{1+\sigma^+}}\, J_o(\lambda) + 2\left(\frac{\omega}{\lambda} - 1\right) Z(\lambda,\sigma^+) \right\} \\
& - \omega\Delta H_o^{(2)}(\omega) \sqrt{\frac{1-\sigma^+}{1+\sigma^+}} + \frac{\hat{u}_d}{W_m} e^{-i\lambda\sigma^+} \gamma_{o_s}(\sigma^+) \\
& + \frac{i\omega\Delta}{\pi} e^{-i\omega}\left[\sum_{-\infty}^{-1} + \sum_{1}^{\infty}\right] e^{in\tau}[\Lambda(\sigma^+,g_c) + \Lambda(\sigma^+,h_c) - \Lambda(\sigma^+,g_1) - \Lambda(\sigma^+,h_1)] \\
& + \frac{\Delta e^{-i\omega}}{\pi}\sqrt{\frac{1-\sigma^+}{1+\sigma^+}}\left[\sum_{-\infty}^{-1} + \sum_{1}^{\infty}\right] e^{in\tau}[E_1 + E_2 - i\omega e^{i\omega}(B_1 + B_2)] \quad . \qquad (2)
\end{aligned}$$

The quantities Δ and $\gamma_{o_s}(\sigma^+)$ are defined as

$$\Delta = -\frac{2\pi e^{i\omega}\{y_m^+\hat{u}_d[J_o(\lambda) - J_2(\lambda) - 2J_1(\lambda)] + \hat{v}_d[J_o(\lambda) - iJ_1(\lambda)]\}}{i\omega\pi e^{i\omega}[H_1^{(2)}(\omega) + iH_o^{(2)}(\omega)] + M_c} ,$$

$$M_c = \left[\sum_{-\infty}^{-1} + \sum_{1}^{\infty}\right] e^{in\tau}[C_1 + C_2 - 2] - i\omega e^{i\omega}\left[\sum_{-\infty}^{-1} + \sum_{1}^{\infty}\right] e^{in\tau}[D_1 + D_2] , \tag{3}$$

and

$$\gamma_{o_s}(\sigma^+) = 2W_m\sqrt{\frac{1-\sigma^+}{1+\sigma^+}}\left\{2y_m^+(1+\sigma^+) + \alpha_m + (y_m^+ + \alpha_m)\frac{\left[\sum_{-\infty}^{-1}+\sum_{1}^{\infty}\right](E_1+E_2)}{2 - \left[\sum_{-\infty}^{-1}+\sum_{1}^{\infty}\right](C_1+C_2-2)}\right\} \tag{4}$$

for a cascade which experiences a nonconvected disturbance of reduced frequency λ. The flow angle, Φ, is related to the stagger angle, ξ, and the mean swirl angle, β, entering the cascade, as $\Phi = \pi - \beta - \xi$.

Equations (2), (3), and (4) contain several quantities expressing the contribution of the blades in a cascade which are adjacent to the reference blade. Each of these quantities are infinite summations and are listed in Table I.

NUMERICAL TECHNIQUES

Several numerical methods were employed to give a solution to the problem described above. They are discussed in the following.

Averaging Procedure

The evaluation of the infinite summations having the form $\left[\sum_{-\infty}^{-1} + \sum_{1}^{\infty}\right] A$ in the present analysis, as listed in Table I, uses the convergence criterion

called "Averaging Procedure." This method was previously used by Henderson [1] in his computer program to calculate some of these unsteady cascade terms. The mathematical expression of the criterion is:

$$\mathrm{DIFF} = \left| \frac{\sum_{3}^{N+1} \left[\sum_{-N}^{-1} + \sum_{1}^{N} \right] A}{N - 2} - \frac{\sum_{3}^{N} \left[\sum_{-N}^{-1} + \sum_{1}^{N} \right] A}{N - 3} \right| < \varepsilon \tag{5}$$

The number of pairs of arguments, N, required for an accurate estimation of these infinite cascade summations can be determined. It was observed that as the value of ε decreases, the number of argument pairs which are summed increases and the summation approaches a finite value in an asymptotic manner. An example is illustrated in Figure 2. The validity of the method is therefore assured by this converging behavior.

Gauss-Legendre Quadrature Method

When evaluating the unsteady lift and pitching moment using the calculated unsteady pressure difference across the airfoil chord, a difficulty arises when performing the integration from the leading to trailing edge due to the existence of a singularity at the leading edge. This difficulty was overcome by the use of the Gauss-Legendre (G-L) method. The Gauss-Legendre quadrature employed has a behavior which is most helpful with the present problem, and, in addition, the very high order interpolating polynomial which is implicit in Gauss quadrature can better approximate the value of integration near the singularity [2]. The region of integration is further divided into two parts such that one of the integrals is carried out over a small region close to the singularity. For example,

$$\tilde{C}_L = \frac{1}{2\pi}\int_{-1}^{1} \Delta\tilde{C}_p(\sigma^+)d\sigma^+$$

$$= \frac{1}{2\pi}\left[\int_{-1}^{-.98} \Delta\tilde{C}_p(\sigma^+)d\sigma^+ + \int_{-.98}^{1} \Delta\tilde{C}_p(\sigma^+)d\sigma^+\right]$$

The theory by Sears for the unsteady response of an isolated flat-plate airfoil [3] was used to check the accuracy of this integration scheme. Excellent agreement was found between the results using the G-L method and those obtained by the exact solution developed by Sears, Figure 3.

COMPUTER PROGRAM

The expression developed for unsteady pressure distribution is introduced above. The Equations (2), (3), and (4) have been programmed using complex Fortran IV language. This program can be run on the P. S. U. 360 system. Note that memory size S = 280K must be used.

The computer program, Appendix A, consists of a main program and several subroutines. The arrangement of this program and its subroutines is as follows:

	MAIN PROGRAM
Subroutines	UPDBC
	CCSUM
	RSUM1
	RSUM2
	BESJ
	BESY
	ZED
	ATANF

The main program also performs the integration required to find unsteady lift and moment. Part of the major effort was to generate the subroutines permitting the evaluation of the infinite summations occurring in the analysis. The subroutines are listed and described below:

SUBROUTINES	INPUT	OUTPUT	DESCRIPTION
UPDBC	$s/c, \xi, y_m$, α_m, ω, λ σ^+	$\Delta\tilde{C}_{p_{u_d}}$ $\Delta\tilde{C}_{p_{v_d}}$	The calculation of unsteady pressure difference at certain locations along the airfoil chord and its components contributed by chordwise and transverse disturbances u_d and v_d, respectively.
CCSUM	$s/c, \xi$, ω, σ^+, ϵ	$\left[\sum_{-N}^{-1} + \sum_{1}^{N}\right]$	Evaluation of the infinite cascade summations having the form in Table I. Types of the summations are controlled by the flag, I, I = 1, 2, 3, ... 7.
RSUM1	ω, ns/c, ξ	$D_1 + D_2$	Evaluation of the integrals representing the contribution due to the wakes shed by nth blades.
RSUM2	ω, ns/c, ξ	$B_1 + B_2$	Evaluation of the integrals representing the contribution due to the wakes shed by nth blades.
BESJ, BESY	ω, m	J_m, Y_m	Determines Bessel functions of the first and second kind, order m and argument ω.
ZED	λ, σ^+	Z	A function resulting from the convected disturbance effect.
ATANF	R, I	θ	Conversion of the vectors into polar coordinates.

Since the present analysis determines the effects of blade-to-blade interactions, the program is also capable of solving the isolated airfoil case. As the value s/c increases, values of the cascade terms listed in Table I are found to decrease. If s/c is large enough, their contributions would become negligibly small. To save computational time, the program automatically treats these unsteady cascade terms as zero when the space-chord ratio, s/c, is set to a number larger than 100.0.

For a factor of $\varepsilon = 0.001$, it takes about ten seconds to perform the calculations for each value of ω. Care must be taken for the cases where values of ω are in the neighborhood of resonance points or are extremely small. In these cases, the computational time requited to obtain a solution becomes significantly long.

PREPARATION OF THE INPUT DATA CARDS AND EXAMPLES

The computer program must be arranged in the order described in the previous section. To activate the unsteady pressure program, the input information must be arranged on four input cards in the following fashion:

COMPUTER SYMBOL	SYMBOL IN ANALYSIS	DESCRIPTION	REMARKS
CARD 1 (3F10.4)			
XC	x_c^+	Dimensionless location of concentrated vortices on neighboring blades.	$x_c^+ = -0.50$ in the analysis.
ERR	ε	Parameter controlling the accuracy of calculations of cascade summations.	$\varepsilon = 0.001$ recommended.
BETA	β	The flow exit angle following front stage cascade.	$\beta = 0°$, if inflow has no swirl. (degrees)
CARD 2 (3I10)			
KRUN		Number of runs to calculate using different combinations of parameters described on Card 3.	
IRUN		Number of runs to calculate using different values of reduced frequencies, ω and λ, which are described on Card 4.	
JRUN		If JRUN = 0, print out $\tilde{C}_L$ and $\tilde{C}_M$ only; = 1, print out complete results including $\Delta\tilde{C}_p$ as a function of x_p^+, $\tilde{C}_L$ and $\tilde{C}_M$.	

CAPD 3 (4F10.4)

SOC	s/c	Space-to-chord ratio of cascade.	If s/c larger than 100.0, the program goes to isolated airfoil calculation.
EPS	ξ	Stagger angle of cascade.	(degrees)
YM	y^{+}_{max}	Dimensionless maximum camber; ratio of maximum camber to the blade half-chord.	
AM	α_m	Mean incidence angle of the flow relative to cascade.	(degrees)

CARD 4 (2F10.4)

OMEGA	ω	Reduced frequency based on the mean flow velocity (convected).
OMEGB	λ	Reduced frequency based on the transport velocity of disturbance over the airfoil (nonconvected).

To illustrate the use of the program listed in Appendix A, the following examples are presented.

Example No. 1

To preform calculations for a given cascade at several values of reduced frequency

$\beta = 0°$, $s/c = 2.209$, $\xi = 45°$, $y_m = 0.05$, $\alpha_m = 1.2°$;

(i) $\omega = 1.0$, $\lambda = 1.0$;

(ii) $\omega = 2.0$, $\lambda = 2.0$;

(iii) $\omega = 3.0$, $\lambda = 3.0$.

The input data cards will be arranged as shown in Figure 4.

Example No. 2

To perform calculations with different values of s/c, ξ, y_m, or α_m, for $\beta = 30°$ and several values of reduced frequency:

(i) $s/c = 2.029$, $\xi = 45°$, $y_m = 0.05$, $\alpha_m = 1.20°$;

(ii) $s/c = 2.029$, $\xi = 55°$, $y_m = 0.05$, $\alpha_m = 3.50°$;

(iii) $s/c = 1.135$, $\xi = 35°$, $y_m = 0.0$, $\alpha_m = 0.0°$; (flat plate airfoil

with nonconvected reduced frequencies of:

(i) $\omega = 1.0$, $\lambda = 0.8$;

(ii) $\omega = 2.0$, $\lambda = 1.7$.

Note, there will be 3 X 2 different cases calculated. The input data cards will be arranged as shown in Figure 5.

Example No. 3

To perform calculation for an isolated airfoil, i.e., neglecting the blade-to-blade interactions, while still including the effects of blade camber and angle of incidence, set s/c to a number larger than 100.0. Consider the case

$\beta = 30°$, $s/c = \infty$, $\xi = 45°$, $y_m = 0.08$, $\alpha_m = -1.2°$

(i) $\omega = 0.50$, $\lambda = 0.40$;

(ii) $\omega = 1.00$, $\lambda = 0.80$.

The input data cards for this example are listed in Figure 6.

References

[1] Henderson, R. E., "The Unsteady Response of an Axial Flow Turbomachine to an Upstream Disturbance," Ph.D. dissertation, Engineering Department, University of Cambridge, Cambridge, England, 1972.

[2] Stroud, A. H. and Secrest, D., "Gaussian Quadrature Formulas," Prentice-Hall, 1966.

[3] Sears, W. R., "Some Aspects of Nonstationary Airfoil Theory and its Practical Application," Journal of the Aeronautical Sciences, Vol. 8, No. 3, Jan. 1941, pp. 104-108.

Table I

Infinite Summations for Unsteady Effect in Cascade

I	CASCADE SUMMATIONS	MAIN PROGRAM Eq. (2)	γ_{o_s} Eq. (3)	Δ Eq. (4)	REMARKS
1	$\left(\sum_{-\infty}^{1} + \sum_{1}^{\infty}\right)(C_1 + C_2 - 2)$		X		REAL
2	$\left(\sum_{-\infty}^{-1} + \sum_{1}^{\infty}\right) e^{in\tau} (C_1 + C_2 - 2)$			X	COMPLEX
3	$\left(\sum_{-\infty}^{-1} + \sum_{1}^{\infty}\right) e^{in\tau}(B_1 + B_2)$	X			COMPLEX
4	$\left(\sum_{-\infty}^{-1} + \sum_{1}^{\infty}\right) e^{in\tau}(D_1 + D_2)$			X	COMPLEX
5	$\left(\sum_{-\infty}^{-1} + \sum_{1}^{\infty}\right) e^{in\tau}\left(\Lambda(\sigma^+,g_c) + \Lambda(\sigma^+,h_c) - \Lambda(\sigma^+,g_1) - \Lambda(\sigma^+,h_1)\right)$	X			COMPLEX
6	$\left(\sum_{-\infty}^{-1} + \sum_{1}^{\infty}\right) (E_1 + E_2)$		X		REAL
7	$\left(\sum_{-\infty}^{-1} \quad \sum^{\infty}\right) e^{in\tau}(E_1 + E_2)$	X			COMPLEX

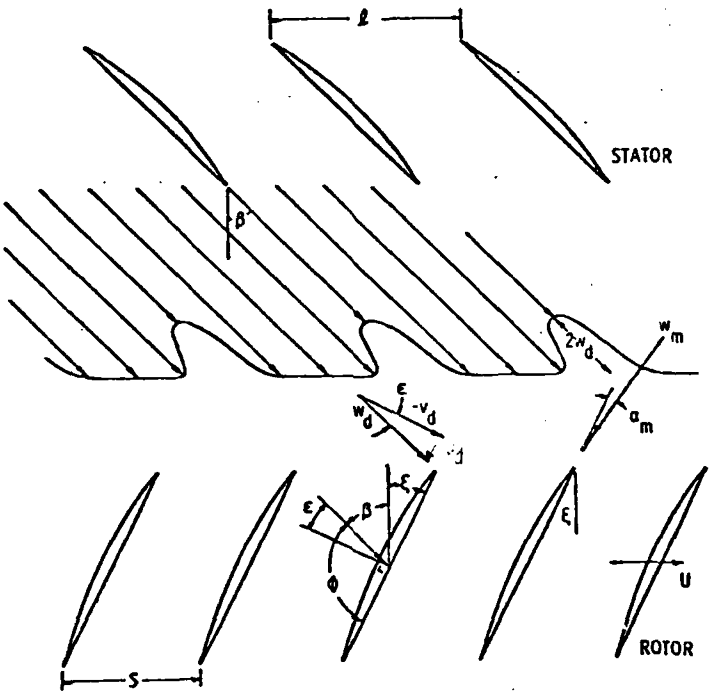


Figure 1. General Disturbance Flow Field

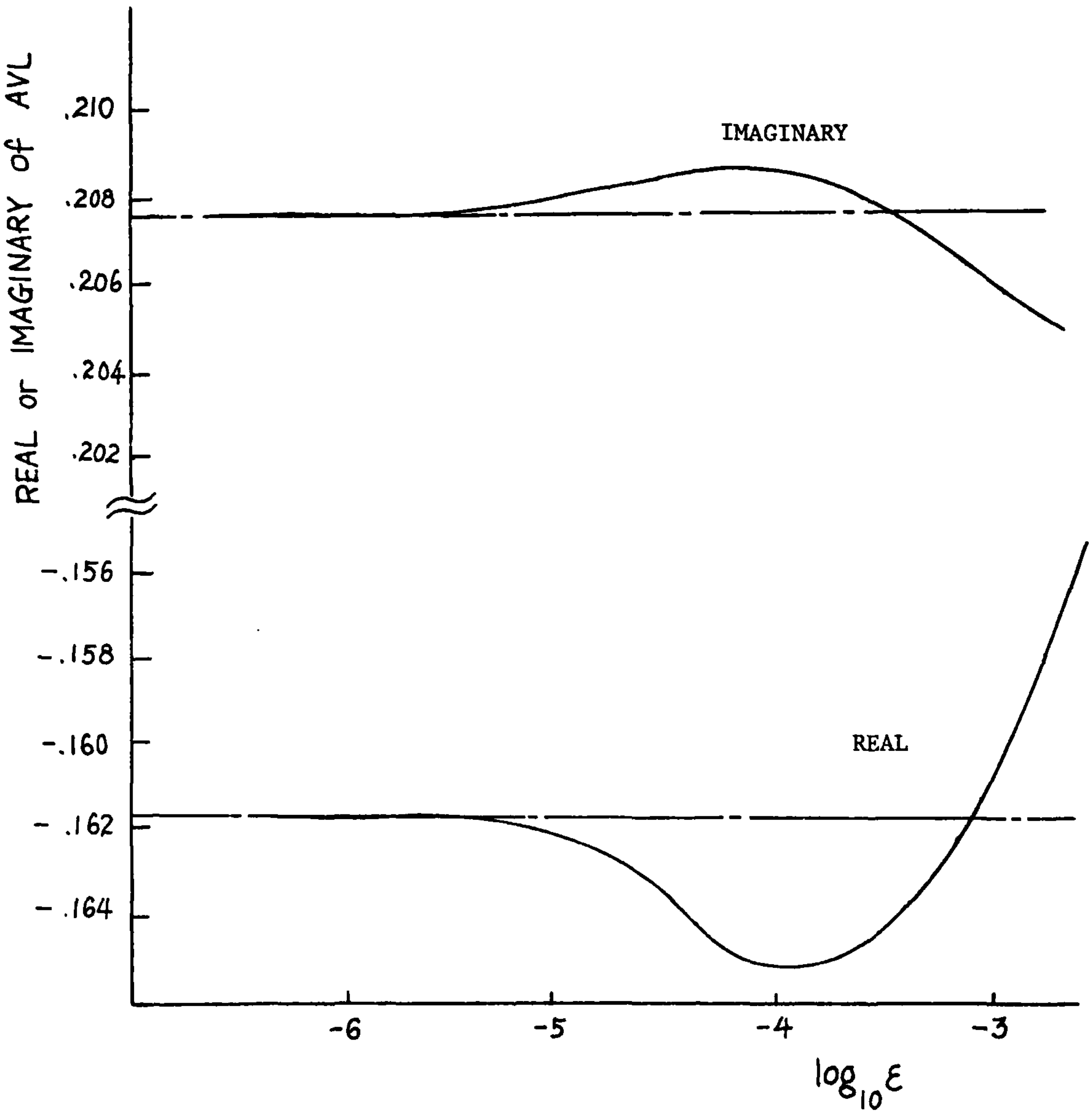


Figure 2. Behavior of AVL = $\left[\sum_{-N}^{-1} + \sum_{1}^{N}\right] e^{in\tau}\,[D_1 + D_2]$ as a function of ε for

$\xi = 45^{o}$, $s/c = 1.0$, $\omega = 1.0$

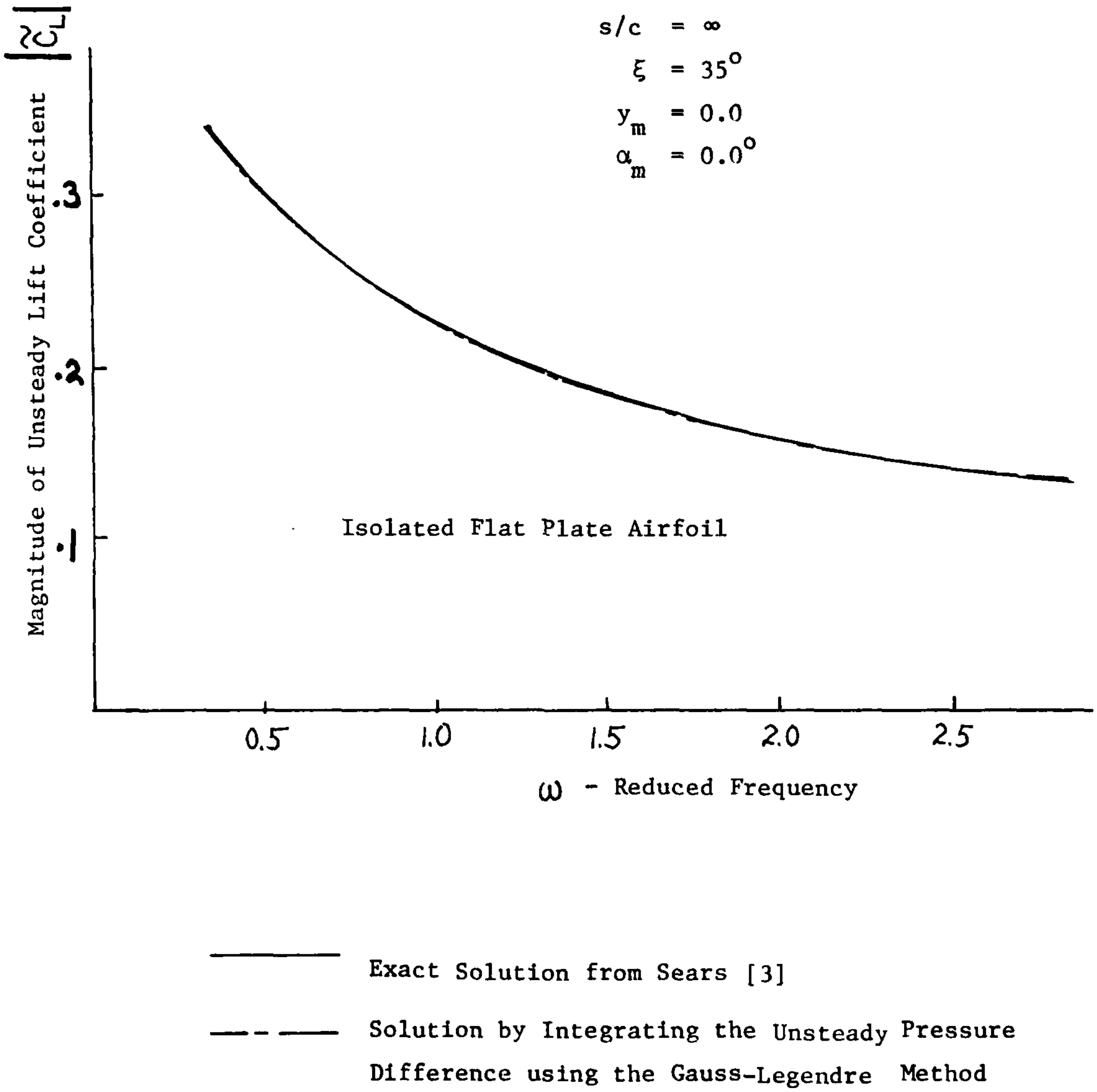


Figure 3. Comparison Between the Exact and the Gauss-Legendre Method

```
//DATA.INPUT DD *
   -0.5000    0.0010     0.0000
         1         3          1
    2.0290   45.0000     0.0500    1.2000
    1.0000    1.0000
    2.0000    2.0000
    3.0000    3.0000
```

Figure 4. Input cards for Example No. 1.

```
//DATA.INPUT DD *
   -0.5000    0.0010    30.0000
         3         2          0
    2.0290   45.0000     0.0500    1.2000
    1.0000    0.8000
    2.0000    1.7000
    2.0290   55.0000    0.0500    3.5000
    1.0000    0.8000
    2.0000    1.7000
    1.1350   35.0000    0.0000    0.0000
    1.0000    0.8000
    2.0000    1.7000
```

Figure 5. Input cards for Example No. 2.

```
//DATA.INPUT DD *
  -0.5000    0.0010   30.0000
        1         2         1
 100.0000   45.0000    0.0800   -1.2000
   0.5000    0.4000
   1.0000    0.8000
```

Figure 6. Input cards for Example No. 3.

```
C     SOURCE LANGUAGE:  FORTRAN IV

C
C     ***************************************************************
C                       UNSTEADY PRESSURE PROGRAM
C                   WRITTEN BY I. C. SHEN     9/9/79
C
C                   CONTENTS
C                                 MAIN PROGRAM
C                SUBROUTINES      UPDBC
C                                 CCSUM
C                                 RSUM1
C                                 RSUM2
C                                 BESJ
C                                 BESY
C                  FUNCTIONS      ZED
C                                 ATANF
C     ***************************************************************
C
C


// EXEC FWCG,PARM='NOSOURCE,ISNS'
//SYSIN DD *
      IMPLICIT REAL*8(A-H,O-Z,$)
      REAL*8 NSC,JK1,JK2,JK3,JK4
      COMPLEX*16 EAVA,EAVB,AVL,AVM,AVP,AVJ,AVK,AVER,
     +           ZED,Z,TPD,PDU,PDV,
     +           HANK0,HANK1,MC,NC,DELU,DELV,DEL,RI,EXW,EXL,
     +           CL,CLU,CLV,CPD,CLWH,CM,CMU,CMV
      DIMENSION XP(24),TPD(24),PDU(24),PDV(24),XI(12),W(12)
      COMMON /AVIV/ TAU,SINXI,COSXI
      COMMON /RSUM/ JK1,JK2,JK3,JK4
      COMMON /MAIN/ EAVA,EAVB,AVL,AVM,AVP,AVJ,AVK,Z,XC,ERR
      COMMON /DELTA/ RJA0,RJA1,RJA2,RY0,RY1,RJB0,RJB1,RJB2,HANK0,HANK1,
     1               MC,NC,DEL,DELU,DELV,RI,EXW,PHI,SINPH,COSPH
C
C     WEIGHTING COEFFICIENTS FOR THE GAUSS-LENGENDRE METHOD
C     M=24 IN EACH INTERVAL OF INTEGRATION.
C
      DATA PI/3.1415927D0/
      DATA XI/0.0640568929D0, 0.1911188675D0, 0.3150426797D0,
     +        0.4337935076D0, 0.5454214714D0, 0.6480936519D0,
     +        0.7401241916D0, 0.8200019860D0, 0.8864155270D0,
     +        0.9382745520D0, 0.9747285560D0, 0.9951872200D0/,
     +      W/0.1279381953D0, 0.1258374563D0, 0.1216704729D0,
     +        0.1155056682D0, 0.1074442701D0, 0.0976186521D0,
     +        0.0861901615D0, 0.0733464814D0, 0.0592985849D0,
     +        0.0442774388D0, 0.0285313886D0, 0.0123412298D0/
C
C    READ DATA
C    *****************************************************************
C    *    SOC---SPACE-CHORD RATIO                                    *
C    *    EPS---STAGGER ANGLE (DEGREES)                              *
C    *    YM ---RATIO OF MAXIMUM CAMBER TO HALF-CHORD                *
C    *    AM ---MEAN FLOW INCIDENCE ANGLE (DEGREES)                  *
C    *    KRUN--NUMBER OF DIFFERENT CASCADE GEOMETRIES               *
C    *      TO BE CONSIDERED.                                        *
C    *    IRUN--NUMBER OF REDUCED FREQUENCIES TO BECONSIDERED.       *
C    *    JRUN = 0  PRINT OUT ONLY THE UNSTEADY LIFT                 *
C    *              AND PITCHING MOMENT.                             *
C    *         = 1  PRINT OUT ALL THE RESULTS.                       *
C    *****************************************************************
15250
```

```
C
      READ(5,14) XC,ERR,BETA
   14 FORMAT(3F10.4)
      READ(5,15) KRUN,IRUN,JRUN
   15 FORMAT(3I10)
      DO 20 K=1,KRUN
      READ(5,10) SOC,EPS,YM,AM
   10 FORMAT(4F10.4)
      WRITE(6,600) SOC,XC,ERR,EPS,BETA,YM,AM
  600 FORMAT(//////////////////
     14X,'S/C=',F10.4,8X,'XCN=',F10.4,8X,'ERR=',E12.4//
     14X,'STAGGER=',F10.4,'(DEG)',8X,'BETA=',F10.4,'(DEG)'//
     14X,'YM=',F10.4,8X,'AM=',F10.4,'(DEG)'///)
      IF(SOC.GE.100.0D0) WRITE(6,103)
      GO TO 104
  103 FORMAT(////8X,'**********  ISOLATED AIRFOIL CASE  **********'//
     120X,'S/C --> 00')
  104 PHI=180.D0-EPS-BETA
      COSPH=DCOS(PHI/57.29547D0)
      SINPH=DSIN(PHI/57.29547D0)
      AM=AM/57.29547D0
      SINXI=DSIN(EPS/57.29547D0)
      COSXI=DCOS(EPS/57.29547D0)
      DO 25 I=1,IRUN
      READ(5,40) OMEGA,OMEGB
   40 FORMAT(2F10.4)
      OMGAB=OMEGA/OMEGB
C     ---PARAMETER TESTING THE PRESENCE OF NON-CONVECTIVE EFFECT.
      C=YM*YM+AM*AM
C     ---PARAMETER TESTING THE PRESENCE OF STEADY VORTICITY
C        [EQN   IN SHEN'S THESIS]
      TAU=-2.*SOC*OMEGA*DCOS(BETA/57.29547D0)/SINPH
      WRITE(6,650) OMEGA,OMEGB,TAU
  650 FORMAT(///2X,'**************************************************
     1********************
     1'//15X,'OMEGA=',F10.4,8X,'LAMDA=',F10.4/15X,'TAU=',F10.4//2X,
     1'*************************************************************
     1******'//)
C
C   *****************************************************************
C     DETERMINE UNSTEADY CIRCULATION COEFFICIENT (DEL) BY
C     EVALUATING CASCADE SUMMATIONS THAT ARE NOT FUNCTIONS
C     OF XP.
C   *****************************************************************
C
      IF(SOC-100.) 102,101,101
  101 EAVA=DCMPLX(0.0D0,0.0D0)
      EAVB=DCMPLX(0.0D0,0.0D0)
      AVL=DCMPLX(0.0D0,0.0D0)
      AVM=DCMPLX(0.0D0,0.0D0)
      GO TO 120
  102 CALL CCSUM(1,OMEGA,0.0D0,XC,SOC,EPS,ERR,EAVA,IAVA)
       EAVB=DCMPLX(0.D0,0.D0)
       IF(C.EQ.0.D0) GO TO 107
      CALL  CCSUM(2,OMEGA,0.0D0,XC,SOC,EPS,ERR,EAVB,IAVB)
  107 CALL  CCSUM(3,OMEGA,0.0D0,XC,SOC,EPS,ERR,AVL,IAVL)
      CALL  CCSUM(4,OMEGA,0.0D0,XC,SOC,EPS,ERR,AVM,IAVM)
  120 RI=DCMPLX(0.0D0,1.0D0)
      EXW=CDEXP(RI*OMEGA)
      EXL=CDEXP(-RI*OMEGB)
      CALL BESJ (OMEGB,0,RJB0,1.0D-5,IE0)
      CALL BESJ (OMEGB,1,RJB1,1.0D-5,IE1)
      CALL BESJ (OMEGB,2,RJB2,1.0D-5,IE2)
      CALL BESJ (OMEGA,0,RJA0,1.0D-5,IA0)
      CALL BESJ (OMEGA,1,RJA1,1.0D-5,IA1)
      CALL BESJ (OMEGA,2,RJA2,1.0D-5,IA2)
```

15250

```
      CALL BESJ (OMEGA,2,RJA2,1.0D-5,IA2)
      CALL BESY (OMEGA,0,RY0,IY0)
      CALL BESY (OMEGA,1,RY1,IY1)
      HANK0=DCMPLX(RJA0,-RY0)
      HANK1=DCMPLX(RJA1,-RY1)
      MC=EAVA-RI*OMEGA*EXW*AVL
      NC=RI*OMEGA*PI*EXW*(HANK1+RI*HANK0)+MC
      DELU=-2.*PI*EXW*(-YM*COSPH*(RJB0-RJB2-RI*2.*RJB1))/NC
      DELV=-2.*PI*EXW*(-SINPH*(RJB0-RI*RJB1))/NC
      DEL=DELU+DELV
C
C    ***************************************************************
C    SEPARATE THE INTERVAL OF INTEGRATION FROM XP+= [-1, 1] INTO
C    XP+ = [-1, -0.98] AND [-0.98, 1] WITH 24 UNEVEN STEPS IN EACH
C    REGION TO GIVE AN ACCURATE ESTIMATION OF THE IMPROPER INTEGRAL.
C    ***************************************************************
C
      M=0
   11 DO 33 J=1,12
          XP(J)=-.99D0-.01D0*XI(13-J)
   33     XP(J+12)=-.99D0+.01D0*XI(J)
          GO TO 222
   12     DO 22 J=1,12
          XP(J)=.01D0-.99D0*XI(13-J)
   22     XP(J+12)=.01D0+.99D0*XI(J)
C
C   *****************************************************************
C     CALCULATION OF THE UNSTEADY PRESSURE DISTRIBUTION AT THE
C     LOCATIONS SPECIFIED BY THE G-L METHOD.
C   *****************************************************************
C
  222 DO 444 J=1,24
       IF(SOC-100.) 202,201,201
  201 AVP=DCMPLX(0.0D0,0.0D0)
      AVJ=DCMPLX(0.0D0,0.0D0)
      AVK=DCMPLX(0.0D0,0.0D0)
      GO TO 205
C
C---- EVALUATE THE CASCADE SUMMATIONS THAT ARE FUNCTIONS
C---- OF XP
C
  202 CALL  CCSUM (5,OMEGA,XP(J),XC,SOC,EPS,ERR,AVP,IAVP)
      CALL  CCSUM (6,OMEGA,XP(J),XC,SOC,EPS,ERR,AVJ,IAVJ)
      AVK=DCMPLX(0.D0,0.D0)
      IF(C.EQ.0.D0) GO TO 205
      CALL  CCSUM (7,OMEGA,XP(J),XC,SOC,EPS,ERR,AVK,IAVK)
  205 Z=ZED(OMEGB,XP(J),ERR)
      CALL UPDBC(TPD(J),PDU(J),PDV(J),OMEGA,OMEGB,XP(J),EPS,SOC,YM,AM,BE
     1TA)
  444 CONTINUE
C
C    ******************************************************************
C     INTEGRATION OF UNSTEADY PRESSURE ALONG THE CHORD TO FIND UNSTEADY
C     LIFT  AND PITCHING MOMENT.
C     ---- BY THE GAUSS-LENGENDRE QUADRATURE METHOD
C    ******************************************************************
C
      IF(M.EQ.1) GO TO 602
      CLU=DCMPLX(0.D0,0.D0)
      CLV=DCMPLX(0.D0,0.D0)
      CMU=DCMPLX(0.D0,0.D0)
      CMV=DCMPLX(0.D0,0.D0)
      DO 26 J=1,12
      A=.01D0*W(13-J)
      CLU=A*PDU(J)+CLU
      CLV=A*PDV(J)+CLV
      CMU=A*PDU(J)*XP(J)+CMU
```

15300

```
   26 CMV=A*PDV(J)*XP(J)+CMV
      DO 28 J=13,24
      B=.01D0*W(J-12)
      CLU=B*PDU(J)+CLU
      CLV=B*PDV(J)+CLV
      CMU=B*PDU(J)*XP(J)+CMU
   28 CMV=B*PDV(J)*XP(J)+CMV
      M=M+1
      GO TO 12
  602 DO 37 J=1,12
      A=.99D0*W(13-J)
      CLU=A*PDU(J)+CLU
      CLV=A*PDV(J)+CLV
      CMU=A*PDU(J)*XP(J)+CMU
   37 CMV=A*PDV(J)*XP(J)+CMV
      DO 39 J=13,24
      B=.99D0*W(J-12)
      CLU=B*PDU(J)+CLU
      CLV=B*PDV(J)+CLV
      CMU=B*PDU(J)*XP(J)+CMU
   39 CMV=B*PDV(J)*XP(J)+CMV
C     ********************************************************************
C        FIND THE COEFFICIENTS OF UNSTEADY LIFT AND PITCHING MOMENT
C        AND LOCATION OF UNSTEADY CENTER-OF-PRESSURE
C     ********************************************************************
      CLU=CLU/(2.D0*PI)
      CLV=CLV/(2.D0*PI)
      CMU=-CMU/(4.D0*PI)
      CMV=-CMV/(4.D0*PI)
      CL=CLU+CLV
      CLWH=CL*EXL
      CLX=DREAL(CL)
      CLY=DIMAG(CL)
      CLMAG=DSQRT(CLX**2+CLY**2)
      PHASE=57.29578D0*ATANF(CLX,CLY)
      CLWX=DREAL(CLWH)
      CLWY=DIMAG(CLWH)
      PHAWH=57.29578D0*ATANF(CLWX,CLWY)
      CM=CMU+CMV
      CMX=DREAL(CM)
      CMY=DIMAG(CM)
      CMMAG=DSQRT(CMX**2+CMY**2)
      PHASM=57.29578D0*ATANF(CMX,CMY)
      XCP=.5D0-CMMAG/CLMAG
      WRITE(6,800) CL,CLMAG,PHASE,CLU,CLV,CLWH,PHAWH,CM,CMMAG,PHASM,CMU,
     +CMV,XCP
  800 FORMAT(/5X,'UNSTEADY LIFT W.R.T. MIDCHORD'/5X,'L" /(2*PI*RHO*W
     1*WD*EXT)'//
     112X,'CL"',12X,'CLMAG',4X,'PHASE(DEG)',11X,'CLU"',14X,'CLV"', 5X,
     1'CL" W.R.T. LEADING EDGE',4X,'PHASE(DEG)'/
     15X,F8.4,',',F8.4,2X,F8.4,4X,F8.4,5X,3(F8.4,',',F8.4,2X),5X,F8.4//
     15X,'UNSTEADY PITCHING MOMENT COEFF. W.R.T. MIDCHORD'/
     15X,' M"/(4.*PI*RHO*W*WD*EXT)'//12X,'CM"',12X,'CMMAG',4X,'PHASE(DEG
     1)',9X,'CMU"',12X,'CMV"'/5X,F8.4,',',F8.4,2X,F8.4,4X,F8.4,5X,2(F8.4
     1,',',F8.4,2X)//10X,'UNSTEADY CENTER-OF-PRESSURE LOCATION'/20X,'XCP
     1/C'=',F8.4)
C
C     ********************************************************************
C       PRINT OUT UNSTEADY PRESSURE DISTRIBUTION, IF JRUN = 1
C     ********************************************************************
C
15300
```

```
      IF(JRUN.EQ.0) GO TO 25
      ANG1=57.29578D0*DATAN(-OMEGB)
      WRITE(6,250)
  250 FORMAT(///4X,'UNSTEADY PRESSURE DISTRIBUTION'/17X,'TPD/(RHO*W*WD)'
     1/5X,'% CHORD',6X,'REAL',6X,'IMAG',6X,'MAG',5X,'PHASE(DEG)',
     12X,'TPU/(RHO*W*WD)',3X,'PUMAG',4X,'TPV/(RHO*W*WD)',3X,'PVMAG',
     11X,'PHASE WRT L.E.'/)
      DO 75 J=1,24
      XOC=50.*(1.+XP(J))
      CPD=TPD(J)
      TPDR=DREAL(CPD)
      TPDI=DIMAG(CPD)
      PMAG=DSQRT(TPDR**2+TPDI**2)
      ANG=57.29578D0*ATANF(TPDR,TPDI)
      ANGL=ANG+ANG1
      PUMAG=CDABS(PDU(J))
      PVMAG=CDABS(PDV(J))
      WRITE(6,350) XOC,CPD,PMAG,ANG,PDU(J),PUMAG,PDV(J),PVMAG,ANGL
  350 FORMAT(4X,F8.4,2X,2(F8.4,2X),F8.4,4X,F8.4,2X,2F8.4,1X,F8.4,1X,2F8.
     14,1X,F8.4,1X,F8.4)
   75 CONTINUE
   25 CONTINUE
   20 CONTINUE
      STOP
      END
15300
```

```
      SUBROUTINE UPDBC (TPD,PDU,PDV,OMEGA,OMEGB,XP,EPS,SOC,YM,AM,BETA)
C    ********************************************************************
C     ACCORDING TO EQUATION (39)   IN SHEN'S THESIS
C     ---- CALCULATION OF UNSTEADY PRESSURE FUNCTION
C    ********************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z,$)
      COMPLEX*16 Z,RI,TMAU,TMAV,TMA,DEL,MC,NC,EXW,DELU,DELV,TMB,TMBU,TMB
     1V,TMCU,TMCV,TMDD,TMDU,TMDV,SVOR,TMEU,TMEV,PDU,PDV,TPD,HANKO,HANK1,
     1EAVA,EAVB,AVL,AVM,AVP,AVJ,AVK
      COMPLEX*16 ZED
      COMMON /MAIN/ EAVA,EAVB,AVL,AVM,AVP,AVJ,AVK,Z,XC,ERR
      COMMON /DELTA/ RJAO,RJA1,RJA2,RYO,RY1,RJBO,RJB1,RJB2,HANKO,HANK1,
     1              MC,NC,DEL,DELU,DELV,RI,EXW,PHI,SINPH,COSPH
      DATA PI/3.1415927DO/
      SXP=DSQRT(1.ODO-XP**2)
      TXP=DSQRT((1.0DO-XP)/(1.0DO+XP))
      OMGAB=OMEGA/OMEGB
      TMAU=-4.*COSPH*YM*((OMGAB-1.)*(2.*XP*Z-SXP*RJBO)-RI*TXP*RJB1-2.*RI
     1      *OMEGA*Z/(OMEGB**2))
      TMAV=-2.*SINPH*(TXP*RJBO+2.*(OMGAB-1.)*Z)
      TMA=TMAU+TMAV
      TMBU=-OMEGA*DELU*HANKO*TXP
      TMBV=-OMEGA*DELV*HANKO*TXP
      TMCU=(RI*OMEGA*DELU*AVP)/(PI*EXW)
      TMCV=(RI*OMEGA*DELV*AVP)/(PI*EXW)
      TMDD=TXP*(AVJ-RI*OMEGA*AVM*EXW)/(EXW*PI)
      TMDU=DELU*TMDD
      TMDV=DELV*TMDD
      SVOR=2.DO*TXP*(2.DO*YM*(1.DO+XP)+AM+(YM+AM)*AVK/(2.DO-EAVB))
      TMEU=-COSPH*SVOR*CDEXP(-RI*OMEGB*XP)
      TME=TMEU
      PDU=TMAU+TMBU+TMCU+TMDU+TMEU
      PDV=TMAV+TMBV+TMCV+TMDV
      TPD=PDU+PDV
      RETURN
      END
C
C
C
      FUNCTION ZED (OMEGB,XP,ERR)
      IMPLICIT REAL*8(A-H,O-Z,$)
      COMPLEX*16 ZED,ZED1,RI,RIK
      PSI=3.1415927DO-DARCOS(XP)
      ZED=DCMPLX(0.0DO,0.0DO)
      RI=DCMPLX(0.0DO,1.00DO)
      K=1
   10 CALL BESJ (OMEGB,K,RJK,0.00001DO,IEK)
      RIK=RI**K
      ZED1=RIK*RJK*DSIN(K*PSI)
      ZED=ZED+ZED1
      ZED1R=DREAL(ZED1)
      ZED1I=DIMAG(ZED1)
      IF(DABS(ZED1R)-ERR) 40,40,30
   40 IF(DABS(ZED1I)-ERR) 20,20,30
   30 K=K+1
      GO TO 10
   20 RETURN
      END
15300
```

```
      SUBROUTINE  CCSUM (I,OMEGA,XP,XC,SOC,EPS,ERR,AVER,ISUM)
C
C   ******************************************************************
C       CALCULATION OF THE UNSTEADY CASCADE SUMMATIONS [TABLE 1 IN
C         SHEN'S THESIS]
C         ---BY THE AVERAGING PROCEDURE.
C         OUTPUT:  AVER,ISUM
C   ******************************************************************
      IMPLICIT REAL*8(A-H,O-Z,$)
      REAL*8 IC,NSC,JK1,JK2,JK3,JK4,EAX,LAX,LBX
      COMPLEX*16 AVERR,AVERS,AVERT,AVER,
     +           TN,RT,GC,HC,TGC,THC,GGC,HHC,GHC,GL,HL,TGL,THL,
     +           ERE,RI
      COMMON /AVIV/ TAU,SINXI,COSXI
      COMMON /RSUM/ JK1,JK2,JK3,JK4
      DATA PI/3.1415927D0/
      AVERR=DCMPLX(0.0D0,0.0D0)
      AVERS=DCMPLX(0.0D0,0.0D0)
      AVERT=DCMPLX(0.0D0,0.0D0)
      TXP=DSQRT((1.0D0-XP)/(1.0D0+XP))
      N=1
   99 TN=DCMPLX(0.0D0,N*TAU)
      RT=CDEXP(TN)
      NSC=N*SOC
      IF(I.EQ.3) GO TO 30
      IF (I.EQ.4) GO TO 40
      BIT=2.*NSC*SINXI
      RC=XC+BIT
      IC=2.*NSC*COSXI
      GC=DCMPLX(RC,-IC)
      HC=DCMPLX(RC,IC)
      TGC=CDSQRT((GC+1.0D0)/(GC-1.0D0))
      THC=CDSQRT((HC+1.0D0)/(HC-1.0D0))
      IF(I-2) 10,10,5
    5 IF(I-6)50,60,60
C---- EAVA       I=1
   10 EAX=TGC+THC-2.0D0
      IF(I.EQ.2) GO TO 20
      AVER=EAX*RT
      GO TO 100
C---- EAVB       I=2
   20 AVER=EAX
      GO TO 100
C---- AVL        I=3
   30 CALL RSUM1(OMEGA,NSC,EPS)
      AVER=DCMPLX(JK1,JK2)*RT
      GO TO 100
C---- AVM        I=4
   40 CALL RSUM2(OMEGA,NSC,EPS)
      AVER=DCMPLX(JK3,JK4)*RT
      GO TO 100
C---- AVP        I=5
   50 EAX=DREAL(TGC+THC)
      LAX=DREAL(TGC*THC)
      ARG=EAX*DSQRT(1.0D0-XP**2)/((1.0D0+XP)-LAX*(1.0D0-XP))
      RL=1.0D0+BIT
      GL=DCMPLX(RL,-IC)
      HL=DCMPLX(RL,IC)
      TGL=CDSQRT((GL+1.0D0)/(GL-1.0D0))
      THL=CDSQRT((HL+1.0D0)/(HL-1.0D0))
      EBX=DREAL(TGL+THL)
      LBX=DREAL(TGL*THL)
      ARL=EBX*DSQRT(1.0D0-XP**2)/((1.0D0+XP)-LBX*(1.0D0-XP))
      AVER=2.0D0*RT*(DATAN((ARG-ARL)/(1.0D0+ARG*ARL)))
      GO TO 100
15300
```

```
C---- AVJ        I=6
   60 GGC=TGC/(GC-XP)
      HHC=THC/(HC-XP)
      GHC=GGC+HHC
      IF(I.EQ.7) GO TO 70
      AVER=GHC*RT
      GO TO 100
C---- AVK        I=7
   70 AVER=GHC
C
C AVERGING PROCEDURES TO FIND THE CONVERGED VALUES OF SUMMATIONS
C
  100 AVER=AVER+AVERS
      AVERS=AVER
      IF(N) 120,110,110
  110 N=-N
      GO TO 99
  120 N=-N
      IF(N-3) 170,140,130
  130 AVERT=AVERR/FLOAT(N-3)
  140 AVERR=AVER+AVERR
      IF(N-3) 170,170,150
  150 ERE=AVERR/FLOAT(N-2)-AVERT
      ERE1=DREAL(ERE)
      ERE2=DIMAG(ERE)
      IF(DABS(ERE1)-ERR) 160,160,170
  160 IF(DABS(ERE2)-ERR) 222,222,170
  170 IF(N.GE.50) GO TO 444
      N=N+1
      GO TO 99
  222 IF(N-10)99,99,444
  444 ISUM=N
      RETURN
      END
```

```
      SUBROUTINE RSUM1 (OMEGA,NSC,GSI)
      IMPLICIT REAL*8 (A-H, O-Z,$)
      REAL*8  NSC,LA,JK1,JK2,JK3,JK4
      DIMENSION X(20),W(20)
      COMMON /AVIV/ TAU,SINXI,COSXI
      COMMON /RSUM/ JK1,JK2,JK3,JK4
   10 NNN=10
      X(1)=-0.9739065285D0
      X(2)=-0.8650633666D0
      X(3)=-0.6794095682D0
      X(4)=-0.4333953941D0
      X(5)=-0.1488743389D0
      X(6)=-X(5)
      X(7)=-X(4)
      X(8)=-X(3)
      X(9)=-X(2)
      X(10)=-X(1)
      X(11)=-1.0D0
      W(1)=0.066713443D0
      W(2)=0.1494513491D0
      W(3)=0.2190863625D0
      W(4)=0.2692667193D0
      W(5)=0.2955242247D0
      W(6)=W(5)
      W(7)=W(4)
      W(8)=W(3)
      W(9)=W(2)
      W(10)=W(1)
      W(11)=0.0D0
   15 PI=3.1415926D0
      GS=GSI*PI/180.0D0
      WM=OMEGA
      F=2.0D0*NSC*DSIN(GS)
      E=2.0D0*NSC*DCOS(GS)
      LAMDA=-1
      TA=0.0D0
      TB=0.0D0
      DO 400  N=1,200
      LAMDA=LAMDA+2
      IF (LAMDA-1) 51,51,52
   51 N4=NNN+1
      GO TO 53
   52 N4=NNN
   53 SA=0.0D0
      SB=0.0D0
      DO 500  M=1,N4
      LA=X(M)+LAMDA+1.0D0
      AL=(LA+F-1.0D0)**2+E**2
      AL1=2.0D0*(LA+F-1.0D0)
      AL2=AL**2
      AL3=AL**3
      AL4=AL**4
      AL5=AL**5
      BE=(LA+F)**2-1.0D0+E**2
      BE1=2.0D0*(LA+F)
      A=BE/AL
      B=2.0D0*E/AL
      H=A**2+B**2
      SQRTH=DSQRT(DABS(H))
      SH1=SQRTH*H
      SH2=SH1*H
      SH3=SH2*H
      C=2.0D0*(A+SQRTH)
      SQRTC=DSQRT(DABS(C))
      SQ1=C*SQRTC
      SQ2=SQ1*C
```

```
      SQ3=SQ2*C
      G=SQRTC-2.0D0
      A1=(AL*BE1-BE*AL1)/AL2
      A2=2.0D0*(AL*(AL-BE1*AL1-BE)+BE*AL1**2)/AL3
      AS=-AL**2*(AL1+BE1)
      AT=AL*(AL1**2*BE1+2.0D0*AL1*BE)
      A3=6.0D0*(AS+AT-BE*AL1**3)/AL4
      B1=-2.0D0*E*AL1/AL2
      B2=4.0D0*E*(AL1**2-AL)/AL3
      B3=12.0D0*E*AL1*(2.0D0*AL-AL1**2)/AL4
      H1=2.0D0*(A*A1+B*B1)
      H2=2.0D0*(A1**2+A*A2+B*B2+B1**2)
      H3=2.0D0*(3.0D0*A1*A2+3.0D0*B1*B2+A*A3+B*B3)
      C1=2.0D0*A1+H1/SQRTH
      C2=2.0D0*A2-0.5D0*H1**2/SH1+H2/SQRTH
      C3=2.0D0*A3+0.75D0*H1**3/SH2-1.5D0*H1*H2/SH1+H3/SQRTH
      G1=0.5D0*C1/SQRTC
      G2=-0.25D0*C1**2/SQ1+0.5D0*C2/SQRTC
      G3=0.375D0*C1**3/SQ2-0.75D0*C1*C2/SQ1+0.5D0*C3/SQRTC
      IF (M-NNN) 61,61,62
   61 A4A=AL2*(-AL+AL1**2+2.0D0*AL1*BE1+BE)
      A4B=AL*AL1**2*(-BE1*AL1-3*BE)
      A4=24.0D0*(A4A+A4B+BE*AL1**4)/AL5
      B4=48.0D0*E*(AL**2-3.0D0*AL*AL1**2+AL1**4)/AL5
      H4=2.0D0*(3.0D0*(A2**2+B2**2)+4.0D0*(A1*A3+B1*B3)+A*A4+B*B4)
      C4A=H1**4/SH3
      C4B=H1**2*H2/SH2
      C4C=H2**2/SH1
      C4D=H1*H3/SH1
      C4=2.0D0*A4-1.875D0*C4A+4.5D0*C4B-1.5D0*C4C-2.0D0*C4D+H4/SQRTH
      G4A=C1**4/SQ3
      G4B=C1**2*C2/SQ2
      G4C=C2**2/SQ1
      G4D=C1*C3/SQ1
      G4=-15.0D0/16.0D0*G4A+2.25D0*G4B-0.75D0*G4C-G4D+0.5D0*C4/SQRTC
      FA=G4*DCOS(WM*LA)
      FB=G4*DSIN(WM*LA)
      SA=SA+W(M)*FA
      SB=SB+W(M)*FB
      GO TO 500
   62 SZ=G
      S1=G1
      S2=G2
      S3=G3
  500 CONTINUE
      TA=TA+SA
      TB=TB+SB
      GA=-SZ*DSIN(WM)/WM-S1*DCOS(WM)/WM**2+S2*DSIN(WM)/WM**3+(S3*DCOS(WM
     1)+TA)/WM**4
      GB=SZ*DCOS(WM)/WM-S1*DSIN(WM)/WM**2-S2*DCOS(WM)/WM**3+(S3*DSIN(WM)
     1+TB)/WM**4
      GB=-GB
   29 CONTINUE
      IF (LAMDA-30) 400,72,72
   72 YN=LAMDA+2.0D0
      IF (OMEGA-0.5D0) 511,76,76
  511 EN=12.0D0/YN**4
C     TESTA AND TESTB ARE MAXIMUM POSSIBLE ERROR BOUNDS OBTAINED FROM
C          ASYMPTOTIC BEHAVIOR OF INTEGRAND
      TESTA=EN/DABS(TA)
      TESTB=EN/DABS(TB)
      LT=LAMDA+2
  601 IF (TESTA-0.02D0) 75,75,400
   75 IF (TESTB-0.02D0) 76,76,400
  400 CONTINUE
   76 GA=-SZ*DSIN(WM)/WM-S1*DCOS(WM)/WM**2+S2*DSIN(WM)/WM**3+(S3*DCOS(WM
     1)+TA)/WM**4
      GB=SZ*DCOS(WM)/WM-S1*DSIN(WM)/WM**2-S2*DCOS(WM)/WM**3+(S3*DSIN(WM)
```

```
     1+TB)/WM**4
      GB=-GB
      JK2=GB
      JK1=GA
  603 RETURN
      END





      SUBROUTINE RSUM2 (OMEGA,NSC,GSI)
      IMPLICIT REAL*8 (A-H, O-Z,$)
      REAL*8  LA,MU,MU1,MU2,MU3,M2,M3,M4,M5,JK1,JK2,JK3,JK4,NSC
      DIMENSION X(20),W(20)
      COMMON /AVIV/ TAU,SINXI,COSXI
      COMMON /RSUM/ JK1,JK2,JK3,JK4
   10 NNN=10
      X(1)=-0.9739065285D0
      X(2)=-0.8650633666D0
      X(3)=-0.6794095682D0
      X(4)=-0.4333953941D0
      X(5)=-0.1488743389D0
      X(6)=-X(5)
      X(7)=-X(4)
      X(8)=-X(3)
      X(9)=-X(2)
      X(10)=-X(1)
      W(1)=0.0666713443D0
      W(2)=0.1494513491D0
      W(3)=0.2190863625D0
      W(4)=0.2692667193D0
      W(5)=0.2955242247D0
      W(6)=W(5)
      W(7)=W(4)
      W(8)=W(3)
      W(9)=W(2)
      W(10)=W(1)
   15 PI=3.1415926D0
C     GSI IS CASCADE STAGGER ANGLE
      GS=GSI*PI/180.0D0
C     OMEGA IS REDUCED FREQUENCY
      WM=OMEGA
C     NSC IS BLADE NUMBER TIMES BLADE SPACING TO CHORD RATIO
      F=2.0D0*NSC*DSIN(GS)
C     NNN IS THE NUMBER OF POINTS USED IN THE G-L QUADRATURE
      E=2.0D0*NSC*DCOS(GS)
      SA=0.0D0
      SB=0.0D0
      IF (F) 26,26,27
   27 LS=1
      GO TO 35
   26 LAMDA=-1
      LS=-F+3
   30 LAMDA=LAMDA+2
      RA=0.0D0
      RB=0.0D0
      DO 310  L=1,NNN
      LA=X(L)+LAMDA+1.0D0
      DEL=(LA+F)**2-1.0D0-E**2
      GAM=2.0D0*E*(LA+F)
      MU=DEL**2+GAM**2
      C=DEL/MU
      D=GAM/MU
      H=C**2+D**2
```

```
      A=2.0D0*(C+DSQRT(DABS(H)))
      RA=RA+DSQRT(DABS(A))*DCOS(WM*LA)*W(L)
      RB=RB+DSQRT(DABS(A))*DSIN(WM*LA)*W(L)
  310 CONTINUE
      SA=SA+RA
      SB=SB+RB
      IF (LAMDA+2-LS) 30,32,32
   32 LS=LAMDA+2
   35 LA=LS
      DEL=(LA+F)**2-1.0D0-E**2
      DEL1=2.0D0*(LA+F)
      GAM=E*DEL1
      GAM1=2.0D0*E
      MU=DEL**2+GAM**2
      MU1=2.0D0*(DEL*DEL1+GAM*GAM1)
      MU2=2.0D0*(DEL1**2+2.0D0*DEL)+8.0D0*E**2
      MU3=12.0D0*DEL1
      C=DEL/MU
      C1=DEL1/MU-DEL*MU1/MU**2
      C2=2.0D0/MU-2.0D0*DEL1*MU1/MU**2-DEL*MU2/MU**2+2.0D0*DEL*MU1**2/MU
     1**3
      C3=3.0D0*(MU**2*(-2.0D0*MU1-DEL1*MU2-4.0D0*DEL*DEL1)+2.0D0*MU*MU1*
     1(DEL1*MU1+DEL*MU2)-2.0D0*DEL*MU1**3)/MU**4
      D=GAM/MU
      D1=GAM1/MU-GAM*MU1/MU**2
      D2=-2.0D0*GAM1*MU1/MU**2-GAM*MU2/MU**2+2.0D0*GAM*MU1**2/MU**3
      D3=(-6.0D0*E*MU2-GAM*MU3)/MU**2+MU1*(12.0D0*E*MU1+6.0D0*GAM*MU2)/M
     1U**3-6.0D0*GAM*MU1**3/MU**4
      H=C**2+D**2
      H1=2.0D0*(C*C1+D*D1)
      H2=2.0D0*(C1**2+D1**2+C*C2+D*D2)
      H3=6.0D0*(C1*C2+D1*D2)+2.0D0*(C*C3+D*D3)
      SQRTH=DSQRT(DABS(H))
      A=2.0D0*(C+SQRTH)
      A1=2.0D0*C1+H1/SQRTH
      A2=2.0D0*C2-0.5D0*H1**2/(H*SQRTH)+H2/SQRTH
      A3=2.0D0*C3+0.75D0*H1**3/(H**2*SQRTH)-1.5D0*H1*H2/(H*SQRTH)+H3/SQR
     1TH
      SQRTA=DSQRT(DABS(A))
      G=SQRTA
      G1=0.5D0*A1/SQRTA
      G2=-0.25D0*A1**2/(A*SQRTA)+0.5D0*A2/SQRTA
      G3=0.375D0*A1**3/(A**2*SQRTA)-0.75D0*A1*A2/(A*SQRTA)+0.5D0*A3/SQRT
     1A
      SN=DSIN(WM*LA)
      CN=DCOS(WM*LA)
      TA=-G*SN/WM-G1*CN/WM**2+G2*SN/WM**3+G3*CN/WM**4
      TB=G*CN/WM-G1*SN/WM**2-G2*CN/WM**3+G3*SN/WM**4
      LAMDA=LS-2
      LF=LS+20
      UA=0.0D0
      UB=0.0D0
      DO 400  N=1,200
      LAMDA=LAMDA+2
      VA=0.0D0
      VB=0.0D0
      DO 500  M=1,NNN
      LA=X(M)+LAMDA+1.0D0
      DEL=(LA+F)**2-1.0D0-E**2
      DEL1=2.0D0*(LA+F)
      GAM=E*DEL1
      GAM1=2.0D0*E
      MU=DEL**2+GAM**2
      M2=MU**2
      M3=M2*MU
      M4=M3*MU
      M5=M4*MU
      MU1=2.0D0*(DEL*DEL1+GAM*GAM1)
```

```
      MU2=2.0D0*(DEL1**2+2.0D0*DEL)+8.0D0*E**2
      MU3=12.0D0*DEL1
      C=DEL/MU
      C1=DEL1/MU-DEL*MU1/M2
      C2=2.0D0/MU+(-2.0D0*DEL1*MU1-DEL*MU2)/M2+2.0D0*DEL*MU1**2/M3
      C3A=M2*(-2.0D0*MU1-DEL1*MU2-4.0D0*DEL*DEL1)
      C3B=2.0D0*MU*MU1*(DEL1*MU1+DEL*MU2)
      C3=3.0D0*(C3A+C3B-2.0D0*DEL*MU1**3)/M4
      C4A=-2.0D0*M3*(MU2+4.0D0*DEL1**2+2.0D0*DEL)
      C4B=M2*(4.0D0*MU1**2+4.0D0*DEL1*MU1*MU2+16.0D0*DEL*DEL1*MU1+DEL*MU
     12**2)
      C4C=MU*MU1**2*(-4.0D0*DEL1*MU1-6.0D0*DEL*MU2)
      C4=6.0D0*(C4A+C4B+C4C+4.0D0*DEL*MU1**4)/M5
      D=GAM/MU
      D1=GAM1/MU-GAM*MU1/M2
      D2=(-2.0D0*GAM1*MU1-GAM*MU2)/M2+2.0D0*GAM*MU1**2/M3
      D3A=(-6.0D0*E*MU2-GAM*MU3)/M2
      D3B=6.0D0*MU1*(2.0D0*E*MU1+GAM*MU2)/M3
      D3=D3A+D3B-6.0D0*GAM*MU1**3/M4
      D4A=-8.0D0*M3*(E*MU3+3.0D0*GAM)
      D4B=M2*(48.0D0*E*MU1*MU2+8.0D0*GAM*MU1*MU3+6.0D0*GAM*MU2**2)
      D4C=-12.0D0*MU*MU1**2*(4.0D0*E*MU1+3.0D0*GAM*MU2)
      D4=(D4A+D4B+D4C+24.0D0*GAM*MU1**4)/M5
      H=C**2+D**2
      H1=2.0D0*(C*C1+D*D1)
      H2=2.0D0*(C1**2+D1**2+C*C2+D*D2)
      H3=6.0D0*(C1*C2+D1*D2)+2.0D0*(C*C3+D*D3)
      H4=6.0D0*(C2**2+D2**2)+8.0D0*(C1*C3+D1*D3)+2.0D0*(C*C4+D*D4)
      SQRTH=DSQRT(H)
      A=2.0D0*(C+SQRTH)
      A1=2.0D0*C1+H1/SQRTH
      A2=2.0D0*C2-0.5D0*H1**2/(H*SQRTH)+H2/SQRTH
      A3A=H1/(H*SQRTH)
      A3=2.0D0*C3+A3A*(0.75D0*H1**2/H-1.5D0*H2)+H3/SQRTH
      A4A=H1**2/(H**2*SQRTH)
      A4=2.0D0*C4+A4A*(-1.875D0*H1**2/H+4.5D0*H2)-(1.5D0*H2**2+2.0D0*H1*
     1H3)/(H*SQRTH)+H4/SQRTH
      SQRTA=DSQRT(DABS(A))
      G4A=A1**2/(A**2*SQRTA)
      G4=G4A*(-15.0D0/16.0D0*A1**2/A+2.25D0*A2)-(0.75D0*A2**2+A1*A3)/(A*
     1SQRTA)+0.5D0*A4/SQRTA
      VA=VA+G4*DCOS(WM*LA)*W(M)
      VB=VB+G4*DSIN(WM*LA)*W(M)
  500 CONTINUE
      UA=UA+VA
      UB=UB+VB
      GA=SA+TA+UA/WM**4
      GB=SB+TB+UB/WM**4
      GB=-GB
   29 CONTINUE
      IF (LAMDA-30) 400,510,510
  510 YN=LAMDA+2.0D0
      IF (OMEGA-0.5D0) 511,76,76
  511 EN=12.0D0/YN**4
      TESTA=DABS(EN/UA)
      TESTB=DABS(EN/UB)
C     TESTA AND TESTB ARE THE UPPER BOUNDS FOR ERROR OBTAINED FROM
C          ASYMPTOTIC BEHAVIOR OF INTEGRAND
      LT=LAMDA+2
  601 IF (TESTA-0.01D0) 75,75,400
   75 IF (TESTB-0.01D0) 76,76,400
  400 CONTINUE
   76 GA=SA+TA+UA/WM**4
      GB=SB+TB+UB/WM**4
      GB=-GB
      JK3=GA
      JK4=GB
  603 RETURN
      END
```

```
      SUBROUTINE BESJ (X,N,BJ,D,IER)
      IMPLICIT REAL*8 (A-H, O-Z,$)
      BJ=0.0D0
      IF (N) 10,20,20
   10 IER=1
      RETURN
   20 IF (X) 30,30,31
   30 IER=2
      RETURN
   31 IF (X-15.0D0) 32,32,34
   32 NTEST=20.0D0+10.0D0*X-X**2/3
      GO TO 36
   34 NTEST=90.0D0+X/2.0D0
   36 IF (N-NTEST) 40,38,38
   38 IER=4
      RETURN
   40 IER=0
      N1=N+1
      BPREV=0.0D0
C         COMPUTE STARTING VALUE OF M
      IF (X-5.0D0) 50,60,60
   50 MA=X+6.0D0
      GO TO 70
   60 MA=1.4D0*X+60.0D0/X
   70 IX=X
      MB=N+IX/4+2
      MZERO=MA
      IF (MA-MB) 80,90,90
   80 MZERO=MB
C         SET UPPER LIMIT OF M
   90 MMAX=NTEST
  100 DO 190  M=MZERO,MMAX,3
C         SET F(M),F(M-1)
      FM1=1.0D-28
      FM=0.0D0
      ALPHA=0.0D0
      IF (M-(M/2)*2) 120,110,120
  110 JT=-1
      GO TO 130
  120 JT=1
  130 M2=M-2
      DO 160  K=1,M2
      MK=M-K
      BMK=2.0D0*DFLOAT(MK)*FM1/X-FM
      FM=FM1
      FM1=BMK
      IF (MK-N-1) 150,140,150
  140 BJ=BMK
  150 JT=-JT
      S=1+JT
  160 ALPHA=ALPHA+BMK*S
      BMK=2.0D0*FM1/X-FM
      IF (N) 180,170,180
  170 BJ=BMK
  180 ALPHA=ALPHA+BMK
      BJ=BJ/ALPHA
      IF (DABS(BJ-BPREV)-DABS(D*BJ)) 200,200,190
  190 BPREV=BJ
      IER=3
  200 RETURN
      END
```

```
      SUBROUTINE BESY (X,N,BY,IER)
C        CHECK FOR ERRORS IN N AND X
      IMPLICIT REAL*8 (A-H, O-Z,$)
      IF (N) 180,10,10
   10 IER=0
      IF (X) 190,190,20
C        BRANCH IF X LESS THAN OR EQUAL 4
   20 IF (X-4.0D0) 40,40,30
C       COMPUTE Y0 AND Y1 FOR X GREATER THAN 4
   30 T1=4.0D0/X
      T2=T1*T1
      P0=((((-0.0000037043D0*T2+0.0000173565D0)*T2-0.0000487613D0)*T2+0.
     10001734 3D0)*T2-0.001753062D0)*T2+0.3989423D0
      Q0=((((0.0000032312D0*T2-0.0000142078D0)*T2+0.0000342468D0)*T2-0.0
     1000869791D0)*T2+0.0004564324D0)*T2-0.01246694D0
      P1=((((0.0000042414D0*T2-0.0000200920D0)*T2+0.0000580759D0)*T2-0.0
     100223203D0)*T2+0.002921826D0)*T2+0.3989423D0
      Q1=((((-0.0000036594D0*T2+0.0000162 2D0)*T2-0.0000398708D0)*T2+0.00
     101064741D0)*T2-0.0006390400D0)*T2+0.03740084D0
      A=2.0D0/DSQRT(X)
      B=A*T1
      C=X-0.7853982D0
      Y0=A*P0*DSIN(C)+B*Q0*DCOS(C)
      Y1=-A*P1*DCOS(C)+B*Q1*DSIN(C)
      GO TO 90
C       COMPUTE Y0 AND Y1 FOR X LESS THAN OR EQUAL TO 4
   40 XX=X/2.0D0
      X2=XX*XX
      T=DLOG(XX)+0.5772157D0
      SUM=0.0D0
      TERM=T
      Y0=T
      DO 70  L=1,15
      IF (L-1) 50,60,50
   50 SUM=SUM+1.0D0/DFLOAT(L-1)
   60 FL=L
      TS=T-SUM
      TERM=(TERM*(-X2)/FL**2)*(1.0D0-1.0D0/(FL*TS))
   70 Y0=Y0+TERM
      TERM=XX*(T-0.5D0)
      SUM=0.0D0
      Y1=TERM
      DO 80  L=2,16
      SUM=SUM+1.0D0/DFLOAT(L-1)
      FL=L
      FL1=FL-1.0D0
      TS=T-SUM
      TERM=(TERM*(-X2)/(FL1*FL))*((TS-0.5D0/FL)/(TS+0.5D0/FL1))
   80 Y1=Y1+TERM
      PI2=0.6366198D0
      Y0=PI2*Y0
      Y1=-PI2/X+PI2*Y1
C        CHECK IF ONLY Y0 OR Y1 IS DESIRED
   90 IF (N-1) 100,100,130
C        RETURN EITHER Y0 OR Y1 AS REQUIRED
  100 IF (N) 110,120,110
  110 BY=Y1
      GO TO 170
  120 BY=Y0
      GO TO 170
C       PERFORM RECURRENCE OPERATIONS TO FIND YN(X)
  130 YA=Y0
      YB=Y1
      K=1
  140 T=DFLOAT(2*K)/X
      YC=T*YB-YA
      IF (DABS(YC)-1.0D36) 145,145,141
  141 IER=3
      RETURN
  145 K=K+1
```

```
      IF (K-N) 150,160,150
  150 YA=YB
      YB=YC
      GO TO 140
  160 BY=YC
  170 RETURN
  180 IER=1
      RETURN
  190 IER=2
      RETURN
      END
```

```
      FUNCTION ATANF (X,Y)
      IMPLICIT REAL*8 (A-H, O-Z,$)
C     ATANF COMPUTES THE ANGLE WITH INITIAL SIDE, THE POSITIVE X-AXIS,
C     AND TERMINAL SIDE, THE LINE CONTAINING THE ORIGIN AND POINT (X,Y)
C     ANGLES ARE IN RADIANS, GREATER THAN OR EQUAL TO ZERO, LESS THAN 2*
  100 FORMAT (1H ,'INDETERMINATE ANGLE',5X,'X =',F10.5,5X,'Y =',F10.5)
      IF (DABS(X)-0.00001D0) 1,2,2
    1 IF (DABS(Y)-0.00001D0) 3,4,4
    3 WRITE (6,100) X,Y
      ATANF=0.0D0
      RETURN
    4 IF (Y) 5,6,6
    5 ATANF=4.7123889D0
      RETURN
    6 ATANF=1.5707963D0
      RETURN
    2 IF (X) 7,8,8
    7 ATANF=3.1415926D0+DATAN(Y/X)
      RETURN
    8 IF (Y) 9,10,10
    9 ATANF=6.2831853D0+DATAN(Y/X)
      RETURN
   10 ATANF=DATAN(Y/X)
      RETURN
      END
```

DISTRIBUTION LIST FOR UNCLASSIFIED TM 80-10 by I-C. Shen, dated January 5, 1980

Commander
Naval Sea Systems Command
Department of the Navy
Washington, DC 20362
Attn: Library
Code NSEA-09G32
(Copy Nos. 1 & 2)

Naval Sea Systems Command
Attn: T. E. Peirce
Code NSEA-63R3
(Copy No. 3)

Naval Sea Systems Command
Attn: A. R. Paladino
Code NSEA-05H1
(Copy No. 4)

Commanding Officer
Naval Underwater Systems Center
Newport, RI 02840
Attn: Library
Code 54
(Copy No. 5)

Commanding Officer
Naval Ocean Systems Center
San Diego, CA 92152
Attn: D. Nelson
Code 2542
(Copy No. 6)

Naval Ocean Systems Center
Attn: M. M. Rieschman
Code 2542
(Copy No. 7)

Commander
David W. Taylor Naval Ship R&D Center
Department of the Navy
Bethesda, MD 20084
Attn: W. K. Blake
Code 1942
(Copy No. 8)

David W. Taylor Naval Ship R&D Center
Attn: T. E. Brockett
Code 1544
(Copy No. 9)

David W. Taylor Naval Ship R&D Center
Attn: R. A. Cumming
Code 1544
(Copy No. 10)

David W. Taylor Naval Ship R&D Center
Attn: J. H. McCarthy
Code 1552
(Copy No. 11)

David W. Taylor Naval Ship R&D Center
Attn: W. B. Morgan
Code 154
(Copy No. 12)

David W. Taylor Naval Ship R&D Center
Attn: M. M. Sevik
Code 19
(Copy No. 13)

Officer-in-Charge
David W. Taylor Naval Ship R&D Center
Department of the Navy
Annapolis Laboratory
Annapolis, MD 21402
Attn: J. G. Stricker
Code 2721
(Copy No. 14)

Defense Technical Information Center
5010 Duke Street
Cameron Station
Alexandria, VA 22314
(Copy Nos. 15 - 26)

The Pennsylvania State University
Applied Research Laboratory
P. O. Box 30
State College, PA 16801
Attn: I-C. Shen
(Copy No. 27)

Applied Research Laboratory
Attn: R. E. Henderson
(Copy No. 28)

Applied Research Laboratory
Attn: F. S. Archibald
(Copy No. 29)

Applied Research Laboratory
Attn: E. P. Bruce
(Copy No. 30)

Applied Research Laboratory
Attn: D. E. Thompson
(Copy No. 31)

Applied Research Laboratory
Attn: Garfield Thomas Water Tunnel Files
(Copy No. 32)